Thermonuclear reaction rates refinement

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***Introduction.*** In order to calculate controlled thermonuclear fusion problems, it is necessary to know how thermonuclear reaction rates  depend on temperature. These rates are obtained from experimentally measured dependencies of reactions cross sections  on energy. Many experimental works have been published [1] but their accuracy is not high. This can be seen from large discrepancy of experimental papers.

In conventional cross sections processing [2], one chooses a priori  dependence with several free parameters. The parameters values are calculated via least squares fit. Reliability of this approach depends on how adequate  is. In practice, satisfactory accuracy can be obtained for  keV though for  keV the results are unsatisfactory. So, it is reasonable to perform new experiments processing with implication of up-to-date mathematical methods.

***Cross sections database.*** We have considered 4 most important reactions: , , , . For them, database [1] provides ~2000 experimental points obtained in ~90 papers. Data of different authors can diverge up to 6 times! So, we have conducted a thorough analysis of these papers and assigned particular accuracy for each experimental point. For the best works, the accuracy was up to 1% and for the worst it was about several hundred percent. For the ease of further processing, all cross sections were divided by Gamow factor which takes into account penetrability of Coulomb barrier. The obtained ratio is referred to S-factor.

***Regularized double period method.*** The double period method [3] allows to approximate non-periodical function with Fourier series and provides good accuracy on the entire approximation interval. This method doesn't imply a priori formulae for . It allows not only to approximate the curve within the experimental interval but also to extrapolate slightly beyond its boundaries. Implication A.N. Tikhonov regularization to this method makes it possible to process curves measured with large experimental errors. Also, there are no non-physical oscillations of the calculated curve.

***Cross sections processing.*** The above described method was applied to mentioned reactions. The approximations have accuracy about ~2% in experimental energy range of 2 keV–14 MeV. Due to features of S-factor, the approximations can be extrapolated to  as constants.

***Reaction rates***  can be found by calculating integral convolution of the obtained S-factor multiplied by Gamow factor with Maxwell velocities distribution. The integration was performed numerically. This provided  as numerical array. For convenience of using the rates in applied calculations, the rates were approximated by double period method with 12 Fourier coefficients. The accuracy of this approximation is about 3% in temperature range 10 eV–2 MeV. The obtained formulae refine known data [2] within 10–20% at low temperatures and drastically at high temperatures.

References

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