BEAM TRACING CALCULATIONS IN PLASMAS BEYOND THE EIKONAL REGIME

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The widely used assumption of a regular wavefield is the eikonal form **E**(**r**) = **E**0(**r**) exp(iϕ(**r**)) with both **E**0 and ∇ϕ being slowly varying functions compared to ϕ. The short-wavelength asymptotic methods such as ray tracing (RT) and, later, beam tracing (BT) [1] were initially based on this approximation. But the microwave propagation in fusion-like plasma often goes beyond the eikonal regime. The most typical cases are (i) sharply curved microwave beams (e.g. at cutoffs), (ii) different absorption rate across the beam (small-angle arrival at the absorbing layer) and (iii) the cusp caustics, where the group velocity becomes zero (LH and UH resonances). In any of these cases, the RT validity is not violated according to the power-flow viewpoint [2]. As for the applicability of BT, the problems (i) and (ii) can be reduced to the sum of eikonal solutions by means of the narrow-beamlet decomposition–tracing–summation, at a relatively low computational cost [3]. The same method can be used in cases where the beam, though being of an eikonal form, rapidly disintegrates due to geometrical spreading. At last, the case (iii), with respect to the BT calculations, is examined theoretically and numerically in this work.

The technique of BT comprises the RT procedure so as to find the beam axis in the (**r**, **k**) hyperspace, next, the computation of the complex-valued Hessian matrix ***Q*** ≡ ∇∇ϕ all along the axis and then, the calculation of the wave amplitude in selected points of the axis. The evolution of ***Q*** is obtained by solving the matrix Riccati equation, which brings to a singular behavior of ***Q*** near the point where the wave power flow is halted, thus forming a cusp on the beam axis. Within some vicinity of such a point, numerical solvers will not be able to provide reliable results. However one can pass through the cusp region by considering the Fourier transform of **E**(**r**). The Gaussian beam (GB) in spectral representation is of a similar form to its spatial one, but with the shifted phase, the inverse phase-Hessian ***Q***‑1 and the amplitude multiplied by the factor {det(2πi***Q***‑1)}1/2. In the *k*-space GB is of an eikonal form near the cusp (on the contrary, it is non-eikonal in uniform media), and the evolution equation for ***Q***‑1 is free from singularities in this region. So the modified BT procedure around the cusp consists in numerical solution of the equation for ***Q***‑1 along with the matrix inversion. After the cusp region is left behind, BT may be returned to the standard workflow. Within the proposed modeling framework, the wavefield of GB near the cusp cannot be obtained otherwise than computing the inverse Fourier transform. However the connection formula for matching the amplitudes of incoming and outgoing parts of GB is the same as in the continuous eikonal regime, except for an additional phase shift. So, at no extra cost, the use of BT technique may be extended onto e.g. the propagation of electron Bernstein waves (EBW) in tokamak or stellarator plasmas, which is an important practical issue.

The appropriate modification of the TRUBA code [4] has been carried out. The new capabilities are demonstrated in a number of numerical analyses, which reproduce provisional patterns of X‑EBW heating and current drive in the sample equilibrium plasma configuration of the T-15 Upgrade tokamak [5].

References

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