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PLASMA EQUILIBRIUM IN NON-AXISYMMETRIC SYSTEMS *)

Sorokina E.A.

National Research Centre "Kurchatov Institute", Moscow, Russia, Sorokina_EA@nrcki.ru

The fundamental problem of the plasma equilibrium in a non-axisymmetric magnetic configuration is discussed. In contrast to the idealized case of axial symmetry described by the canonical Grad-Shafranov equation, which has various solutions including analytical and numerical examples of equilibrium systems with toroidally nested magnetic surfaces, the existence of such magneto-plasma systems in the absence of symmetry is not only not obvious, but has been constantly questioned for more than 60 years of development theories of plasma equilibrium.

The urgency of the problem is explained by the absence of symmetrical magnetic systems in reality. Magnetic field asymmetry is inevitable in tokamaks due to the discreteness of magnetic coils, the presence of heating and diagnostic ports, and external fields. Typical approaches for describing such systems by methods of perturbation of initially symmetric configurations encounter the natural difficulty of violating the necessary equilibrium condition – the condition of constancy of the equivalent lengths of force lines on rational magnetic surfaces [1]. Formally, this is manifested in the appearance of resonant denominators in the magnetic differential equation [2,3] describing the closure of currents, which means the presence of singular currents on rational surfaces (see, for example, [4]). The non-physicality of such solutions is interpreted as an inevitable violation of the nesting of magnetic surfaces and the formation of an island structure [1].

Computer modeling does not solve the problem either. Modern 3D numerical codes either admit the presence of singular current layers in the area of rational magnetic surfaces or bypass this area by artificially introducing "shelves" on the profiles of plasma pressure, rotational transformation or a combination thereof.

It should be noted that there is no mathematical restriction on the existence of configurations that ensure the fulfillment of the current closure condition on rational magnetic surfaces, however, within the framework of standard simplified models with the geometry of a flat layer or cylinder, such solutions cannot be built [5]. The absence of illustrative examples of smooth solutions of equilibrium equations with no spatial symmetry has for many years served as an argument in support of the hypothesis by Grad about the non-existence of non-degenerate three-dimensional plasma equilibria with nested magnetic surfaces [6].

In this paper, a class of analytical solutions of equilibrium equations with true magnetic surfaces satisfying the condition of current closure is presented. A convenient system of equilibrium equations [7] is used that generalizes the Grad-Shafranov approach to the case of non-axisymmetric magnetic configurations. The inapplicability of a simple model of a circular cylinder with identified ends for predictive conclusions about the equilibrium of plasma with toroidal topology is revealed and clearly demonstrated. The developed formalism proves the fallacy of the Grad's hypothesis and opens up opportunities for adequate modeling of three-dimensional equilibrium plasma configurations.

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